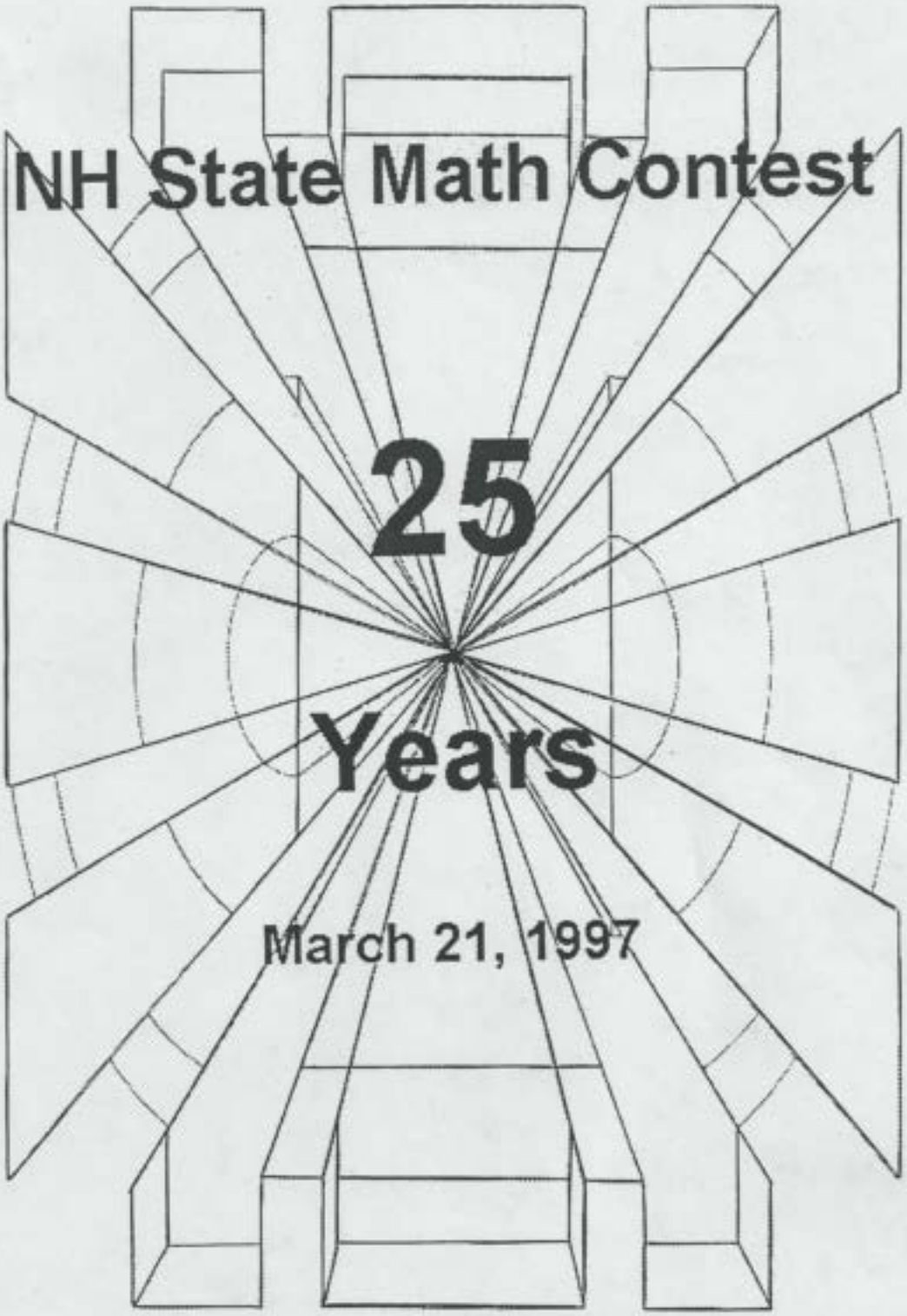
NH State Math Contest



25 Years

March 21, 1997

NHTM NH State Mathematics Contest

In the spring of 1972 the president of NH-ATMNE, Nelson Aldrich of the Claremont Vocational-Technical College, sent out the call to all New Hampshire high schools to form a ten person mathematics team. The reason? We were going to have a statewide mathematics contest!

On the 13th of May in 1972, a beautiful Saturday morning, teams of students from seventeen public and private schools in the state met at Concord High School and participated in an exciting and challenging competition, the very first NH-ATMNE State Mathematics Contest. There has been a contest each year since then except in 1974 when a national gas shortage limited extracurricular activities and transportation of students in many schools.

Since that lively spring day 25 years ago close to ten thousand New Hampshire students have been contestants in our state competitions. Over one thousand dedicated high school and college mathematics teachers have coached students, written questions, proctored rooms of contestants, scored and checked papers, questioned and judged answers, tallied results and congratulated the participants! Through the years the contest rules and categories have changed. Who, in 1972, would have conceived of the idea of using a calculator in a math competition? What has remained constant in the past 25 years is the unflagging interest of NH-ATMNE and the mathematics departments of the USNH in sponsoring the competitions and in bringing students together to foster their enjoyment, learning, and study of mathematics.

On the following pages is a copy of the first state mathematics contest. The contest (questions) were written by George Koehler of Newport High School and typed and duplicated by the secretary to the state math consultant Ferd Prevost. (No computers and laser printers then!) Concord High Math Department Chairperson Arthur Jackson and his staff hosted the event.

The 17 participating schools included:

Bishop Guertin	Manchester Central	Portsmouth	Winnisquam
Concord	Merrimack	St. Paul's	
Dover	Newfound	St. Thomas	
Hanover	Oyster River	Stevens	
Laconia	Phillips Exeter	Timberlane	

The rules for selecting the team members were the same as they were on March 21, 1997 (25th year). The categories, all timed for ten minutes, were Arithmetic, Algebra, Geometry, Advanced Math and Team. Six students participated in each of the first four categories and three in the team category. No one could compete in more than three categories, including the team category. Each of the students in the team category received one-third of the team score for individual points. Questions were valued at two, three, and five points respectively. Thus, the total possible points for each of the first four categories was 60, and the maximum for the team was 30.

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At the conclusion of the first contest, fourteen students, with scores ranging from 22 to 27 points, received CRC Tables as prizes.

Trophies were awarded to the top scoring teams from Laconia High School, St. Paul's School and Phillips Exeter Academy. Their scores were 163, 156, and 145 points respectively.

NHTM NH State Mathematics Contest

1972 NEW HAMPSHIRE STATE MATHEMATICS CONTEST
Sponsored by the New Hampshire Association
of Teachers of Mathematics

ARITHMETIC (CATEGORY 1)

1. (2 pts.) $\left[\dfrac{(.33\overline{3}...)\ (.66\overline{6}...)\ (1.25)}{.2\overline{2}...)\ (.25)\ (650\% - 1.5)}\right] =$

Ans. _____

2. (3 pts.) Pipes A and B can fill a tank in two hours and three hours respectively. Pipe C can empty it in five hours. If all were turned on when the tank is empty, how long will it take to fill the tank?

Ans. _____

3. (5 pts.) A man is hired at a salary of $7,800 a year and gets paid every two weeks. The following deductions are made:

22½% for income tax

2½% for social security

$2.50 for life insurance per check

$3.25 for hospitalization insurance per check

5% for investment in company stock

What is his <u>annual</u> take-home pay? (Figure 52 weeks in a year).

Ans. _____

ALGEBRA (CATEGORY 2)

1. (2 pts.) Which is the greatest and which the least?

(a) $\log(2 + 4)$

(b) $\log 2 + \log 4$

(c) $\log(6-3)$

(d) $\log 6 - \log 3$

(e) $\log 6/\log 3$

Ans.

Greatest _____

Least _____

2. (3 pts.) Which of the following statements are true:

Assume that denominators are non-zero

(a) $|-3| = -3$

(b) $3^2 \cdot 3^3 = 3^6$

(c) $x + y - 3(z + w) = x + y - 3z + w$

(d) $3a + 4b = 7ab$

(e) $3x^{-1} = \frac{1}{3x}$

(f) $\sqrt{x^2 + y^2} = x + y$

(g) $\frac{x + y}{x + z} = \frac{y}{z}$

(h) $\frac{1}{x-y} = \frac{-1}{x+y}$

(i) $\frac{x}{y} + \frac{r}{s} = \frac{x+r}{y+s}$

(j) $\sqrt{-x} \cdot \sqrt{-y} = \sqrt{xy}$

Ans. _____

3. (5 pts.) What are the roots of

$x^4 - 8x^3 + 16x^2 - 8x + 15 = 0$

When it is known that one of its roots is $-i$?

Ans. _____

GEOMETRY (CATEGORY 3)

1. **(2 pts.)** Match the following geometric terms correctly:

_____ (a) Point of concurrence formed by the altitudes of a triangle.

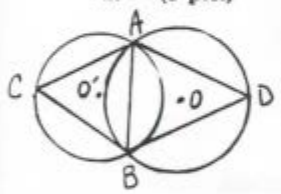
_____ (b) Point of concurrence formed by the medians of a triangle.

_____ (c) Point of concurrence formed by the angle bisectors of a triangle.

_____ (d) Point of concurrence formed by the perpendicular bisectors of the sides of a triangle.

(1) incenter	(2) apothem	(3) Pythagorean triple
(4) centroid	(5) circumcenter	(6) asymptote
(7) orthocenter	(8) locus	

2. **(3 pts.)** Given circle 0 and circle 0′ with the common chord $\overline{AB}$, chord $\overline{AC}$ of circle 0 tangent to circle 0′ at A, chord $\overline{AD}$ of circle 0′ tangent to circle 0 at A, and chords $\overline{BC}$ and $\overline{BD}$ are not in a straight line. If $\overline{BC} = 10''$, and $\overline{BD} = 25''$, find AB.



Hint: Use Similar Triangles

Ans. _____

3. **(5 pts.)** A cathedral tower 200 feet high is 250 feet from a church tower 150 feet high. On the top of each tower is a pigeon. The two pigeons fly off at the same time and at the same speed directly to the same grain on the level, straight road between the towers. The pigeons reached the grain at the same instant. How far is the grain from the foot of the cathedral tower?

Ans. _____

ADVANCED MATH (CATEGORY 4)

1. (2 pts.) If $x + yi = r(\cos\theta + i\sin\theta)$ and $r = \sqrt{x^2 + y^2}$, $x = r\cos\theta$, $y = r\sin\theta$, $\arctan\frac{y}{x} = \theta$ then $-\sqrt{3} - i = ?$

Ans. _____

2. (3 pts.) If $\{G: f(x) = \frac{2}{x+5}\}$ then $\{G^{-1}: \ ? \ \}$

Ans. _____

3. (5 pts.) Match the following equations correctly with the classifications at the right:

1. $2x^2 + 3y^2 - 4x + 6y + 4 = 0$	A. Parabola
2. $y = \frac{3}{x}$	B. Circle
3. $2x = 3y$	C. Point Circle
4. $5x^2 + 5y^2 - 2x - 3y + 6 = 0$	D. Ellipse
5. $7x^2 - 8y^2 - x + 2y - 8 = 0$	E. Equilateral Hyperbola
6. $y^2 - 2y = 4x$	F. Straight Line
7. $x^2 + y^2 = 0$	G. Hyperbola

Ans. 1. _____
2. _____
3. _____
4. _____
5. _____
6. _____
7. _____

TEAM PROBLEMS MISCELLANEOUS (CATEGORY 5)

1. (2 pts.)

$$\frac{\dfrac{\left[\sqrt[3]{8^2} + \text{Sin } 30^\circ)\ (\log_2 128)\ \ (3)^{1/4}\right]}{\left[\text{Cos } (\arctan \sqrt{3})\ (\text{Sin } 240^\circ)\ \ (\frac{\sqrt{3}}{2})^{-1}\right]}}{\left[\frac{1}{\sqrt{3}} - 2\sqrt{3}\right]\ \ \left[\frac{\text{Sin } 90^\circ + \text{Cos } 90^\circ}{5}\right]}$$

Ans. _____

2. (3 pts.) A helicopter shuttle service operating between an airport and the center of a city charges a fare of $10 per person and carries 300 persons per day. The manager estimates that he will lose 15 passengers for each increase of $1.00 for the fare. Find the most profitable fare for him to charge.

Ans. _____

3. (5 pts.) There is given a set S and a binary operation * on the elements of S which are governed by the following properties which makes a group.

1. If a, bϵS, then a * bϵS (closure)
2. If a,b,c,ϵS then (a * b) * c = a * (b * c) (associative)
3. There is an element e of S with the property that for each aϵS, a * e = e * a = a (identity)
4. For each aϵS there is an element $a^{-1}\epsilon$S such that a * a^{-1} = a^{-1} * a = e (inverses)

Does the following operation table define a group?

*	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	b	a
d	d	c	a	b

Ans. Yes or No (Circle one)

if Yes Ans.

if No Ans.

If Yes; what is the identity, also state the inverse for each element of S.

If No; state why not.

Questions submitted by the UNH Mathematics Department.

1972 State Math Contest Answers

Arithmetic

1. 1
2. $\frac{30}{19}$
3. $5310.50

Algebra

1. greatest : e

   least : d
2. none
3. 3, 5, i, -i

Geometry

1. a) 7

   b) 4

   c) 1

   d) 5
2. $5\sqrt{10}$
3. 90°

Advanced Math

1. $2 \operatorname{cis} \frac{7\pi}{6}$
2. $\frac{2-5X}{X}$
3. 1. D

   2. E

   3. F

   4. B

   5. G

   6. A

   7. C

Team

1. 336
2. 15
3. yes

   Identity: a

   $a^{-1} = a$

   $b^{-1} = b$

   $c^{-1} = d$

   $d^{-1} = c$

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From teaching experience we recognize that scores cannot be compared from contest to contest, yet it is interesting to see how close results were in some years! Tests, of course, have varied in the level of difficulty. The point values of questions and numbers of categories have changed. The team category in the 25th year involved all ten members while in the past years the number ranged from three to five students.

TOTAL POINTS POSSIBLE ON CONTESTS

1972-1973	270 points	1975-1987	330 points
1988-1991	360 points	1992-1997	432 points

ACKNOWLEDGEMENTS (from 25th year report)

Our sincere appreciation to Ellen Berchtold of Pinkerton Academy for her work in preparing the cover, to Maureen LeClair of Tim Kurtz's office and to Laconia High School student Dawn Perry and staff members Ruth Stuart and Mina Ayers for doing all the computer and copy work necessary. Thank you so much for all your time and fine work!

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In 1991 Kay Reardon retired from teaching but continued to coordinate the State Math Contest through the 25th contest on March 21, 1997. A big celebration was held on that date at Dover HS! Dave Kent then agreed to coordinate the 26th contest and has served in that capacity until 2013. Prior to that date David Kent and Barbara Hill, as well as many others, had always assisted in this endeavor. Dave always did the contest scoreboards and Barbara was in charge of preparing the certificates for high-scoring participants.

David Kent retired from coordinating the State Math Contest in 2013 and has passed the reins onto Stephen Latvis.

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